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Gravitational Couplings of the Inflaton in Extended Inflation

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Abstract

We discuss a new extended inflationary scenario evading the difficulties of the original model. Our model can thermalize the energy in the bubble walls by the necessary epoch, and establish a Robertson-Walker frame in the bubble clusters. The essential new ingredient in our model is the observation that the coupling of inflaton to the Jordan-Brans-Dicke field is expected to be different from that of visible matter.

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Recently, La and Steinhardt[1] proposed a new inflationary universe scenario, dubbed *extended inflation*, which allows the old inflation model of Guth[2] to succeed in percolating the true vacuum phase. It is based upon a Jordan–Brans–Dicke (JBD)[3] theory coupled to an inflaton field, whose potential admits both a metastable state and a true ground state, separated by a potential barrier. Unfortunately the original extended inflation scenario suffered from some serious flaws, as pointed out by Weinberg[4] and by La, Steinhardt, and Bertschinger.[5] The problems found were twofold: While the true vacuum phase did indeed percolate, there were still problems of too many large bubbles of the new phase whose interiors could not be thermalized in time. There was also a problem with establishing a common Robertson–Walker frame in the various bubble clusters which would eventually coalesce to form our universe.

In this Letter, we will show that these problems with extended inflation can be avoided in a new class of models promulgated by Damour, Gibbons, and Gundlach (DGG).[6] They start with a generalized JBD model in which the JBD scalar field Φ couples with different strengths to "visible" matter and to "invisible" matter (thus leading to a violation of the weak equivalence principle). We will follow the line taken by DGG, and assume that the inflaton of extended inflation has an "invisible" coupling to the JBD scalar field. Since the identity of the inflaton is unknown, there is no reason to believe that it should couple to the JBD scalar field in the same way as does normal matter. Although this may seem ad hoc, it should be emphasized that such a situation in fact arises naturally in superstring theories,[7] where the dilaton plays the role of the JBD scalar field. It is the existence of a new parameter, namely the ratio of the couplings of visible matter and the inflaton field to the JBD scalar field Φ, that allows us to evade the bounds on the ω parameter in JBD models.

We write the action for our theory in the conformal frame in which the visible sector couples only to the metric $g_{\mu\nu}$, and not to the JBD field. Following the metric and

Riemann tensor conventions in Ref.(8), the action becomes:

$$S[g_{\mu\nu}, \Phi, \psi_V, \psi_I] = S_{BD}[g_{\mu\nu}, \Phi] + S_V[g_{\mu\nu}, \psi_V] + S_I[g_{\mu\nu}, \Phi, \psi_I] \tag{1}$$

where $\psi_{V,I}$ denotes the field content of the visible and inflaton sectors in a schematic way. The Brans–Dicke gravitational action $S_{BD}[g_{\mu\nu}, \Phi]$ is given by

$$S_{BD}[g_{\mu\nu}, \Phi] = \int d^4x \sqrt{-g} \left[-\Phi R + \omega g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \Phi}{\Phi} \right], \tag{2}$$

where ω is the JBD parameter constrained by observations[10] to be greater than 500. It was this constraint that led to the problems found in Refs.(4,5) with the original extended inflation picture, since the analysis of the bubble distribution at the end of inflation led to *upper* bounds on ω ($\omega \lesssim 20$ or so), which are in conflict with the aforementioned observational *lower* bound. That upper bounds should exist is not surprising, since if ω is taken to infinity, the theory just becomes the old inflationary model, which is known to have a "graceful exit" problem.[9]

We now suppose that visible matter is described as usual via a perfect fluid stress-energy tensor and will play no role in inflation. Denoting the inflaton field by ψ_I, its action can be written as:

$$\begin{aligned} S_I[g_{\mu\nu}, \Phi, \psi_I] \;\; = \;\; & \int d^4x \sqrt{-g} \left[\frac{1}{2} (16\pi G_N \Phi)^{1-\beta} g^{\mu\nu} \partial_\mu \psi_I \partial_\nu \psi_I \right. \\ & \left. - (16\pi G_N \Phi)^{2(1-\beta)} V(\psi_I) \right], \end{aligned} \tag{3}$$

where $\beta \equiv \beta_I / \beta_V$ and $\beta_{V,I}$ are the couplings of the dilaton in DGG's analysis to the visible and inflaton sectors. β_V is related to ω by $\omega \equiv ({\beta_V}^{-2} - 6)/4$. Solar system tests of Brans–Dicke theories yield the constraint $\omega > 500$, which implies $\beta_V < 0.022$, while present observations are consistent with $|\beta_I| \simeq 1$.[6] In $S_I[g_{\mu\nu}, \Phi, \psi_I]$, $V(\psi_I)$ is the potential for the inflaton and is assumed to be of the standard form for inducing a first-order phase transition via bubble nucleation.

Let us now write the metric in the standard Robertson-Walker form with scale factor $a(t)$. We will assume, as required for "extended" (as well as "old") inflation to occur, that during inflation $\psi_I = \psi_0$, and $V(\psi_0) = \rho_F$, where the energy density of the false vacuum, ρ_F, dominantes the total energy density. Setting $\Lambda = 8\pi G_N \rho_F$, we have the following equations of motion for $a(t)$ and $\Phi(t)$:

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{\Lambda}{3}(16\pi G_N \Phi)^{1-2\beta} + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 - \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} \\
\frac{\ddot{\Phi}}{\Phi} + 3\frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} &= \frac{4\beta\Lambda}{2\omega+3}(16\pi G_N \Phi)^{1-2\beta},
\end{aligned} \tag{4}$$

These equations are most easily solved in terms of the dimensionless field $\chi \equiv 16\pi G_N \Phi$. This system of equations admits power law solutions for $k = 0$, just as in the original extended inflation scenario:

$$\begin{aligned}
a(t) &= a(0)(1+Bt)^p; \qquad p = (\omega - \beta + 3/2)/[(2\beta-1)\beta] \\
\chi(t) &= \chi(0)(1+Bt)^q; \qquad q = 2/(2\beta - 1).
\end{aligned} \tag{5}$$

Here $t = 0$ signifies the onset of inflation, and B is given by

$$B^2 = \frac{4\Lambda\beta^2(2\beta-1)^2[\chi(0)]^{1-2\beta}}{(2\omega+3)(6\omega+9-4\beta^2)}. \tag{6}$$

It can easily be seen that the above results reduce to those found in Ref.(1) when $\beta = 1$. This is as it should be, since in this case, the conformal transformation that takes us from DGG's original action to ours will act on the visible and inflaton sectors identically.

We now turn to the questions of whether inflation occurs in this model and whether a "graceful exit" can be achieved. Clearly, a necessary condition for inflation to occur in our theory is that the exponent p be greater than one so that $\ddot{a}/a$ is positive definite during the vacuum energy dominated period. This first constraint can be written in terms of ω and β as

$$\omega + 3/2 > 2\beta^2. \tag{7}$$

In order that sufficient inflation occur, we require $a(t_{\rm end})/a(0) > 10^{27}$, where $t_{\rm end}$ denotes the end of the inflationary period. Following the analysis of Ref.(4), we relate $a(t_{\rm end})/a(0)$ to $\Phi(t_{\rm end})/\Phi(0)$ via $a(t_{\rm end})/a(0) = [\Phi(t_{\rm end})/\Phi(0)]^{p/q}$. Since $\Phi(t_{\rm end}) \sim M_{Pl}^2$, and neglect of quantum gravity effects requires $\Phi(0) > \rho_F^{1/2} \sim M^2$, we arrive at our second constraint:

$$\omega + 3/2 > \left[1 + \frac{54}{10 - 2\log(M/10^{14}{\rm GeV})}\right]\beta. \tag{8}$$

Next we turn to the constraints coming from percolation and thermalization of the phase transition. The basic techniques for calculating tunnelling amplitudes in field theories were developed by Callan and Coleman,[11] and require computation of the Euclidean action of the bounce configuration that interpolates between the true and false vacuua. The tunnelling rate per unit four-volume is then given by $\lambda = A \ \exp[-S_E(\psi_B)]$, where A is the prefactor containing information about fluctuations about the bounce configuration, and $S_E(\psi_B)$ is the Euclidean action for this configuration (denoted by ψ_B).

This analysis was extended to include the effects of (classical) gravity by Coleman and De Luccia.[12] Unfortunately, their analysis is not directly applicable to our case due to the existence of the JBD field. In order to compute the tunnelling rate in this case, we need to understand how to generalize the existing formalism to the case in which the false vacuum is "rolling" due to the evolution of the Φ field during the bounce. An attempt along these lines was pursued by Accetta and Romanelli[13] with some success. However, for our purposes, we follow previous work of ours on the subject[8] and note that at late times (or equivalently, for large values of the JBD field), the variations of Φ can be neglected, and we can compute λ using the analysis of Ref.(8).

The Euclidean action for the ψ_I field in the truncated theory is given by

$$S_E(\psi_I) = \int d^4x \left[\frac{1}{2}\chi^{1-\beta}(\partial_\mu\psi)^2 + \chi^{2(1-\beta)}V(\psi_I)\right]. \tag{9}$$

As per our analysis in Ref.(8), the formalism of Ref.(11) can be applied to find a χ-independent bounce action. However, when proper care is taken in normalization and in projection of the functional determinant to the subspace orthogonal to the translational zero modes, a χ dependence appears in the pre-factor:

$$\lambda(t) = \lambda_0\ \chi^{2(1-\beta)} = \lambda_0\,[\chi(0)]^{2(1-\beta)}(1+Bt)^{4(1-\beta)/(2\beta-1)}, \tag{10}$$

where λ_0 is the (constant) tunnelling rate for $\chi = 1$. Thus, the physical bubble nucleation rate per unit four-volume is *time dependent* in this theory. This is quite unlike the case in the original extended inflation model at late times.[8,13]

The parameter controlling the percolation properties of the phase transition in this theory is[9] $\epsilon \equiv \lambda(t)/H^4(t)$, where $H(t)$ is the Hubble parameter $\dot{a}(t)/a(t)$. In our model, we have:

$$\epsilon(t) = \frac{\lambda_0\chi(0)^{2(1-\beta)}}{p^4B^4}(1+Bt)^{4\beta/(2\beta-1)}. \tag{11}$$

In order for the nucleation to be successful, clearly $\epsilon(t)$ must increase in time. This implies that $4\beta/(2\beta-1) > 0$, which in turn implies that either $\beta < 0$ or $\beta > 1/2$. This will be subsumed by other constraints. The time, $t_{\rm end}$, for which $\epsilon(t)$ is larger than some critical value of order unity corresponds to the end of the inflationary period.[1,4,9]

We now turn to the constraint coming from the requirement that the bubble clusters that will comprise the observable universe have enough time to thermalize their energy. The point is that the typical bubble cluster consists of a large bubble, together with much smaller ones. Most of the energy of the bubble is tied up in the bubble walls, and collisions with other bubbles are required in order to allow this energy to spread through the bubble interior. The question is how long does it take for this energy to become thermalized, so as to lead to a homogeneous and isotropic universe.

If too many large bubbles are still completing the thermalization process at cosmologically sensitive times, severe conflicts with Big-Bang predictions will clearly ensue. Thus, we must demand that the fraction of space in such bubbles be less than some predetermined value when the tempereature is T. The volume fraction $\mathcal{V}_>(r, t_{\rm end})$, the fraction of the volume contained in bubbles greater than a given (comoving) size r at the end of inflation, can be calculated exactly as in Ref.(4): $\mathcal{V}_>(r, t_{\rm end}) \simeq \ln[p^{-1}(t_{\rm end})](r_0/r)^{\bar\delta}$. Here r_0 is the asymptotic comoving size of a bubble nucleated at $t_{\rm end}$, $\bar\delta \equiv 4\beta^2/(\omega + 3/2 - 2\beta^2)$, and $p(t)$ is the probability of finding a point in space in the false vacuum at time t: $p(t) \sim \exp[-c(t/t_{\rm end})^{(p-1)\bar\delta}] \sim \exp[-c(r_0/r)^{\bar\delta}]$, where c is a constant of order unity.

Imposing the condition that $\mathcal{V}_>(r, t_{\rm end})$ be less than 10^{-n} when the temperature is T, we arrive at the constraint:

$$\omega + 3/2 < \left\{2 + \frac{4[23 + \log_{10}(M/10^{14}{\rm GeV}) + \log_{10}({\rm eV}/T)]}{n + \log_{10}\{\ln[p^{-1}(t_{\rm end})]\}}\right\} \beta^2, \tag{12}$$

It is not unreasonable to suppose that $p(t_{\rm end}) < 1/e$ as in Ref.(4), so that for $M \sim 10^{14}$ GeV and $n \simeq 5$ at recombination ($T \simeq 1/3$ eV), we have the constraint $\omega + 3/2 < 20.7\,\beta^2$. Note that setting $\beta = 1$ we recover the results (and constraints) of Ref.(4). The result is that whereas before the limit was $\omega \lesssim 20$, the limit now is $\omega/\beta^2 \lesssim 20$, which can easily be satisfied for $\omega > 500$.

Finally we turn to the question of reestablishing a common Robertson–Walker frame in all the bubble clusters that will coalesce to form our universe. We must require that there be some way for the system to remember the original (pre bubble-nucleation) coordinates. Weinberg[4] argues that such a record can be found in the time evolution of $a(t)$, or equivalently $\Phi(t)$. Since constant Hubble parameter and $\Phi(t)$ corresponds to the de Sitter situation with *no* distinguished frame, we must require that there be sufficient variations of these quantities. The relevant time interval over which these variations should take place is between $t_{\rm end}$ and $t(r_{\rm univ})$, the time when bubbles with asymptotic sizes equal to that of the observed universe were nucleated. The actual interval may in fact be shorter since we expect that homogeneity and isotropy must hold by the time of recombination or perhaps even nucleosynthesis. Using the fact that $H(t)/H(t_{\rm end}) = (r_{as}(t)/r_0)^{1/(p-1)} \simeq (M/T)^{p-1}$, if we require a variation of m orders of magnitude in the Hubble parameter $H(t)$, we arrive at our final constraint:

$$\omega + 3/2 < \beta(2\beta-1)\left\{1 + m^{-1}[23 + \log_{10}(M/10^{14}{\rm GeV}) + \log_{10}({\rm eV}/T)]\right\} + \beta. \quad (13)$$

Taking $M \sim 10^{14}$ GeV, $T \sim 10^2$ keV, and $m = 1$ as in Ref.(4) leads to the constraint $\omega + 3/2 < 19\beta(2\beta - 1) + \beta$.

We plot the constraints given by Eqs.(7, 8, 12, 13), together with the constraint that $\omega > 500$ in Fig.(1). It should be clear from this plot that there is ample room for all these constraints to be satisfied. If we make the constraints tighter (i.e., demanding that a smaller fraction of space still be thermalizing at recombination, or more orders of magnitude variation in $H(t)$), larger values of β will be required.

To conclude then, we have constructed a model of extended inflation in which the inflaton couples to the JBD field differently than standard visible matter. This yields a theory in which the JBD field is massless (i.e. no potential for Φ is required) and which meets all the requirements for an acceptable inflationary model with *no* fine-tuning! If this avenue is to be pursued further, a realistic particle physics model with the required couplings must be constructed.

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Figure Caption

Fig 1: The allowed region in $\omega - \beta$ space is indicated. Curves 1, 2, 3, 4 correspond to the constraints in Eqs.(7, 8, 12, 13). The observational limit $\omega = 500$ is indicated.